Unity in Diversity

Government Polytechnic, Ahmedabad

Science and Humanities Department

Applied Mathematics

Subject Code: 4320001

Unit -03 Integration and Its Applications

[Marks – 14]

Course Outcome (CO c):

Demonstrate the ability to solve engineering related problems based on applications of integration.

Important formula of Integration

(1) $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + c \; ; n \neq 1$
(2) $\int 1 \, dx = x + c$
(3) $\int \dfrac{1}{x} \, dx = \log_e x + c$
(4) $\int k \, dx = k\,x + c$; where k *is* constant
(5) $\int e^x \, dx = e^x + c$
(6) $\int a^x \, dx = \dfrac{a^x}{\log_e a} + c$
(7) $\int \sin x \, dx = -\cos x + c$
(8) $\int \cos x \, dx = \sin x + c$
(9) $\int \tan x \, dx = \log\lvert \sec x \rvert + c$
(10) $\int \cot x \, dx = \log\lvert \sin x \rvert + c$
(11) $\int \operatorname{cosec} x \, dx = \log\lvert \operatorname{cosec} x - \cot x \rvert + c$ $= \log\left\lvert \tan \dfrac{x}{2} \right\rvert + c$
(12) $\int \sec x \, dx = \log\lvert \sec x + \tan x \rvert + c$ $= \log\left\lvert \tan\left(\dfrac{\pi}{4} + \dfrac{x}{2} \right) \right\rvert + c$

(13)	$\int \sec^2 x \, dx = \tan x + c$
(14)	$\int \operatorname{cosec}^2 x \, dx = -\cot x + c$
(15)	$\int \sec x . \tan x \, dx = \sec x + c$
(16)	$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$
(17)	$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} x + c \;\; or \;\; -\cos^{-1} x + c$
(18)	$\int \frac{1}{\sqrt{1 - x^2}} \, dx = \sin^{-1} x + c \quad or \quad -\cos^{-1} x + c$
(19)	$\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$
(20)	$\int \frac{1}{1 + x^2} \, dx = \tan^{-1} x + c \quad or \quad -\cot^{-1} x + c$
(21)	$\int \frac{1}{\lvert x \rvert \sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \sec^{-1} \frac{x}{a} + c \quad or \quad \frac{-1}{a} \operatorname{cosec}^{-1} \frac{x}{a} + c$
(22)	$\int \frac{1}{x\sqrt{x^2 - 1}} \, dx = \sec^{-1} x + c \quad or \quad -\operatorname{cosec}^{-1} x + c$
(23)	$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \log \left\lvert x + \sqrt{x^2 + a^2} \right\rvert$
(24)	$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \log \left\lvert x + \sqrt{x^2 - a^2} \right\rvert$
(25)	$\int \frac{1}{x^2 - a^2} \, dx = \frac{1}{2a} \log \left\lvert \frac{x - a}{x + a} \right\rvert + c \quad ; \left(x^2 > a^2\right)$
(26)	$\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \log \left\lvert \frac{a - x}{a + x} \right\rvert + c \quad ; \left(x^2 < a^2\right)$

$(27)\ \int \left[f(x) \right]^n f'(x)\ dx = \dfrac{\left[f(x) \right]^{n+1}}{n+1} + c \quad ; n \neq -1$
$(28)\ \int \dfrac{f'(x)}{f(x)}\ dx = \log\left| f(x) \right| + c$

Working rules of Integration

$(29)\ \int \left[f(x) + g(x) \right]\ dx = \int f(x)\, dx + \int g(x)\, dx$
$(30)\ \int \left[f(x) - g(x) \right]\ dx = \int f(x)\, dx - \int g(x)\, dx$
$(31)\ \int k.f(x)\ dx = k \int f(x)\ dx + c;$ k is constant
Integration by Parts $(32)\ \int u \cdot v\ dx = u \int v\ dx - \int \left(\dfrac{du}{dx} \int v\, dx \right) dx + c$

$(33)\ \int \sqrt{x^2 + a^2}\ dx = \dfrac{x}{2} \sqrt{x^2 + a^2} + \dfrac{a^2}{2} \log\left| x + \sqrt{x^2 - a^2} \right| + c$
$(34)\ \int \sqrt{x^2 - a^2}\ dx = \dfrac{x}{2} \sqrt{x^2 - a^2} + \dfrac{a^2}{2} \log\left| x + \sqrt{x^2 - a^2} \right| + c$
$(35)\ \int \sqrt{a^2 - x^2}\ dx = \dfrac{x}{2} \sqrt{a^2 - x^2} + \dfrac{a^2}{2} \sin^{-1} \dfrac{x}{a} + c \quad ; (a > 0)$
$(36)\ \int e^x \left[f(x) + f'(x) \right]\ dx = e^x f(x) + c$

(37) $\int e^{ax} \sin bx \; dx$	$= \dfrac{e^{ax}}{a^2+b^2}\left(a \sin bx - b \cos bx\right)+c$
(37) $\int e^{ax} \cos bx \; dx$	$= \dfrac{e^{ax}}{a^2+b^2}\left(a \cos bx + b \sin bx\right)+c$

Important Trigonometric Substitution

No.	Integrand	Substitution
1	$a^2 - x^2$ or $\sqrt{a^2 - x^2}$	$x = a \sin\theta$ or $x = a\cos\theta$
2	$x^2 - a^2$ or $\sqrt{x^2 - a^2}$	$x = a\sec\theta$ or $x = a\,\mathrm{cosec}\,\theta$
3	$x^2 + a^2$ or $\sqrt{x^2 + a^2}$	$x = a\tan\theta$ or $x = a\cot\theta$
4	$\sqrt{a+x}$	$x = a\tan^2\theta$ or $x = a\cos 2\theta$
5	$\sqrt{a-x}$	$x = a\sin^2\theta$ or $x = a\cos 2\theta$
6	$\sqrt{\dfrac{a-x}{a+x}}$	$x = a\cos 2\theta$
7	$\sqrt{2ax - x^2}$	$x = 2a\sin^2\theta$

Important Trigonometric Formula

Identity No. 1: $\sin^2\theta+\cos^2\theta=1$

$\rightarrow \sin^2\theta=1-\cos^2\theta$

$\rightarrow \cos^2\theta=1-\sin^2\theta$

Identity No. 2: $\tan^2\theta+1=\sec^2\theta$ (We get it by divding $\cos^2\theta$ to Identity No.1)

$\rightarrow \tan^2\theta=\sec^2\theta-1$

$\rightarrow \tan^2\theta-\sec^2\theta=1$

Identity No. 3: $1+\cot^2\theta=\operatorname{cosec}^2\theta$ (We get it by divding $\sin^2\theta$ to Identity No.1)

$\rightarrow \cot^2\theta=\operatorname{cosec}^2\theta-1$

$\rightarrow \operatorname{cosec}^2\theta-\cot^2\theta=1$

Properties of Definite Integration

Even Function:	Odd Function:
If $f(-x)=f(x)$; *for all* x *e.g.* $(1)\cos x,(2)\sec x$ $(3)\,x^2\ (4)\,x^4\ (5)\,x^{(\text{even number})}$	*If* $f(-x)=-f(x)$; *for all* x *e.g.* $(1)\sin x,(2)\tan x\ (3)\operatorname{cosec}x,(4)\cot x$ $(5)\,x\quad(6)\,x^3\quad(7)\,x^{(\text{odd number})}$

Even Function $\times$ *Even Function* $=$ *Even Function*

Odd Function $\times$ *Even Function* $=$ *Odd Function*

Even Function $\times$ *Odd Function* $=$ *Odd Function*

Odd Function $\times$ *Odd Function* $=$ *Even Function*

e.g.

$x^2\cos x$ *is an even function*

$x\cos x$ *is an odd function*

$x^2\sin x$ *is an odd function*

$x\sin x$ *is an even function*

$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_{0}^{a} f(x)\,dx; & \text{if } f(x) \text{ is an even function} \\ 0; & \text{if } f(x) \text{ is an odd function} \end{cases}$
$\int_{a}^{b} f(x)\,dx = \int_{0}^{b} f(a+b-x)\,dx$
$\int_{0}^{a} f(x)\,dx = \int_{0}^{a} f(a-x)\,dx$; (In above formula if $a=0$ and $b=a$)
$\int_{0}^{2a} f(x)\,dx = \int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(2a-x)\,dx;$

Assignment - 3

Unit - 03

Integration and Its Applications:

* Question set for 01 Mark:

1. $\int 6x^5\,dx$ $\underline{x^6 + C}$

$\int 6x^5$

$6\left\{\frac{x^{5+1}}{5+1}\right.$

$\frac{\cancel{6}x^6}{\cancel{6}}$

x^6

(2) $\int \cos^2 x + \sin^2 x\,dx = \underline{x + c}$

→ a) x

$(\because \cos^2 x + \sin^2 x = 1)$

$(\because \int 1\,dx = x + c)$

(3) $\int \frac{\cos x}{\sin x}\,dx = \underline{-\operatorname{cosec}^2 x + c}$

→ $-\operatorname{cosec}^2 x$

(4) $\int a^x\,dx = \frac{a^x}{\log_e a} + c$

→ $\frac{a^x}{\log_e a} + c$

5) $\int \frac{1}{1+x^2} dx = \underline{\tan^{-1}x + c}$

→ c) $\tan^{-1}x$

⑥ $\int \frac{\log x}{x} dx = \underline{\frac{1}{2}(\log x)^2}$

→ $\frac{1}{2}(\log x)^2$

$(\because \log x = t$

$\frac{1}{x} = \frac{dt}{dx}$

$\frac{dx}{x} = dt$

$\int t \cdot dt = \frac{t^2}{2} + c$

$= \boxed{\frac{1}{2}(\log x)^2 + c}$

⑦ $\int_2^5 x^2 dx = \underline{39} + c$

→ ⓐ 39

$\int_2^5 x^2 dx = \left[\frac{x^3}{3}\right]_2^5$

$= \frac{(5)^3}{3} - \frac{(2)^3}{3}$

$= \frac{135}{3} - \frac{8}{3}$

$= \frac{125-8}{3}$

$= \frac{117}{3} = \boxed{39}$

⑧ $\int_0^1 \frac{2}{1+x^2} dx = \underline{\pi/2} + c$

→ ⓑ $\pi/2$

$\underline{\text{sol}^n}$ $2\int\left[\frac{1}{1+x^2}\right]_0^1$

$2[\tan^{-1}x]_0^1$

$2[\tan^{-1}1 - \tan^{-1}0)$

$2(\pi/4 - 0)$

$\boxed{\pi/2}$

9) $\int_0^2 \frac{1}{4+x^2}\,dx = \underline{\pi/8} + C$

$\rightarrow \frac{\pi}{8}$

$\underline{\text{sol}^n}$ $\int_0^2 \frac{1}{x^2+2^2}\,dx$

$= \left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= \frac{1}{2}\cdot\left[\tan^{-1}1 - \tan^{-1}0\right]$

$= \frac{1}{2}\left[\pi/4 - 0\right]$

$= \boxed{\pi/8}$

10) $\int_{-5}^{5} x^3\,dx = \underline{0} + c$

$\rightarrow$ (b) 0

$\underline{\text{sol}^n} = \left[\frac{x^4}{4}\right]_{-5}^{5}$

$= \frac{(5)^4}{4} - \frac{(-5)^4}{4}$

$= \frac{625}{4} - \left(\frac{625}{4}\right)$

$= \frac{0}{4} = 0$

* Question Set for 03 marks :

(1) $\int \frac{x^2+5x+6}{x^2+2x}\,dx$

$\underline{\text{sol}^n}$

$\int \frac{(x+3)(x+2)}{x(x+2)}\,dx$

$= \int \frac{x+3}{x}\,dx$

$\int \frac{x}{x} + \frac{3}{x}$

$= \int 1\,dx + 3\int \frac{1}{x}\,dx$

$= \boxed{x + 3\log|x| + c}$ Ans

② $\int (4x^3 - \frac{1}{x} + \sin x - e^x)\, dx$

Sol^n $= \int (4x^3 - \frac{1}{x} + \sin x - e^x)\, dx$

$= \int 4x^3\, dx - \int \frac{1}{x}\, dx + \int \sin x\, dx - \int e^x\, dx$

$= \cancel{4} \cdot \frac{x^4}{\cancel{4}} - \log|x| + (-\cos x) - e^x$

$= \boxed{x^4 - \log|x| - \cos x - e^x + c}$ Ans

③ $\int x \cdot (2x^2 + 3)^8\, dx$

Sol^n

Let $\frac{d}{dx}$ $2x^2 + 3 = u$

$\frac{d}{dx}$ $2(2x) + 0 = \frac{du}{dx}$

$4x = \frac{du}{dx}$

$x\,dx = du/4$

Now $\int x(2x^2+3)^8\, dx$

$= \int u^8 \cdot du/4$

$= \int \frac{1}{4} u^8\, du$

$= \frac{1}{4} \int u^8\, du$

$= \frac{1}{4} \cdot \frac{u^9}{9} + c$

$= \frac{u^9}{36} + c$

$= \boxed{\frac{(2x^2+3)^9}{36} + c}$ Ans

worship.

④ $\int \frac{(1+x)e^x}{\sin^2(xe^x)}$

Sol^n

Let $xe^x = u$

$x \cdot \frac{d}{dx} e^x + e^x \frac{d}{dx} x = \frac{dy}{dx}$

$= x \cdot e^x + e^x (1) = \frac{dy}{dx}$

$= xe^x + e^x = \frac{dy}{dx}$

$= e^x(1+x)\,dx = du$

Now $\int \frac{dy}{\sin^2 u}$

$= \int \frac{1}{\sin^2 u}\, du$

$= \int \operatorname{cosec}^2 u + c$

$= \cancel{\operatorname{cosec}^2 t}$

$= -\cot u + c$

$= \boxed{-\cot(xe^x) + c}$ Ans

Q-5 $\int e^{\tan x} \sec^2 x\, dx$

Sol^n Let, $\tan x = u$

$\sec^2 x\, dx = du$

Now

$\int e^u \cdot du$

$= e^u + c$

$= \boxed{e^{\tan x} + c}$ Ans

Q.6 $\int \frac{3x+2}{2x^2+x+1}\,dx$

Ans) $\int \frac{3x+2}{2x^2+x+1}\,dx$

$= \int \frac{\frac{3}{4}(4x+1)+\frac{5}{4}}{2x^2+x+1}\,dx$ $\quad \left\{ \int \frac{F'(x)}{F(x)}\,dx = \log|F(x)|+C \right\}$

$= \frac{3}{4}\int \frac{4x+1}{2x^2+x+1}\,dx + \frac{5}{4}\int \frac{1}{2x^2+x+1}\,dx$

$= \frac{3}{4}\log|2x^2+x+1| + \frac{5}{4}\int \frac{1}{2(x^2+x/2+1/2)}\,dx$

$= \frac{3}{4}\log|2x^2+x+1| + \frac{5}{8}\int \frac{1}{x^2+x/2+1/2}\,dx$

$= \frac{3}{4}\log|2x^2+x+1| + \frac{5}{8}\int \frac{1}{(x+1/4)^2-1/16+1/2}$

$= \frac{3}{4}\log|2x^2+x+1| + 5/8\int \frac{1}{(x+1/4)^2+7/16}$

Let $x+1/4 = t$

$= 3/4\log|2x^2+x+1| + 5/8\int \frac{1}{(\sqrt{7}/4)^2+t^2}$

$$= \frac{3}{4}\log|2x^2+x+1| + \frac{5}{8}\cdot\frac{1}{\sqrt{7}/4}\tan^{-1}\left(\frac{x}{\sqrt{7}/4}\right)$$

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$$

$$= \frac{3}{4}\log|2x^2+x+1| + \frac{5}{8_2}\cdot\frac{4}{\sqrt{7}}\tan^{-1}\left(\frac{x+1/4}{\sqrt{7}/4}\right)$$

$$= \frac{3}{4}\log|2x^2+x+1| + \frac{5}{2\sqrt{7}}\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)$$

$$= \boxed{\frac{3}{4}\log|2x^2+x+1| + \frac{5}{2\sqrt{7}}\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)+c}$$ Ans

Q-7 $\int x\cdot e^{3x}\,dx$

Solⁿ $\left\{\int u\cdot v\,dx = u\cdot\int v\,dx - \int\left[\frac{du}{dx}\cdot\int v\,dx\right]dx\right\}$

Let $u = x$, $v = e^{3x}$

$$= x\cdot\int e^{3x}dx - \int\left[1\cdot\int e^{3x}\right]dx$$

$$= x\cdot\frac{e^{3x}}{3} - \int\left[1\cdot\frac{e^{3x}}{3}\right]dx$$

$$= \frac{xe^{3x}}{3} - \frac{1}{3}\int e^{3x}\,dx$$

$$= x\frac{e^{3x}}{3} - \frac{1}{3}\frac{e^{3x}}{3}$$

$$= \boxed{\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c} \quad \text{Ans}$$

Q-8 $\int x \tan^{-1}x \, dx$

Let $u = \tan^{-1}x$, $v = x$

$$\int u \cdot v \, dx = u \cdot \int v dx - \int \left[\frac{du}{dx} \cdot \int v dx\right] dx$$

$$= \tan^{-1}x \int x \, dx - \int \left[\frac{d}{dx}\tan^{-1}x \int x \, dx\right] dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \int \left[\frac{1}{1+x^2} \cdot \frac{x^2}{2}\right] dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{x^2}{1+x^2} dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{(1+x^2)-1}{(1+x^2)} dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left[\int 1 - \frac{1}{1+x^2} dx\right]$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left(x - \tan^{-1}x\right)$$

$= \frac{x^2}{2} \tan^{-1}x - \frac{x}{2} + \frac{1}{2} \tan^{-1}x + c$

$= \frac{1}{2}(x^2+1) \tan^{-1}x - \frac{x}{2} + c$

$= \boxed{\frac{1}{2}(x^2+1) \tan^{-1}x - \frac{x}{2} + c}$ Ans

Q-9 $\int_0^1 \frac{x}{x+1}\, dx$

$= \int_0^1 \frac{x+1-1}{x+1}\, dx$

$= \int_0^1 \frac{(x+1)-1}{x+1}\, dx$

$= \int_0^1 \left(1 - \frac{1}{x+1}\right) dx$

$= [x - \log|x+1|]_0^1$ $\int \frac{1}{x}\, dx = \log|x| + c$

$= [1 - \log|1+1|] - [0 - \log|0+1|]$

$= [1 - \log 2] - [-\log 1]$

$= 1 - \log 2 + \log 1$ $(\because \log 1 = 0)$

$= \boxed{1 - \log 2}$ Ans

Q-10

Q-10 $\int_0^1 \frac{(\log x)^3}{x} dx$

$= \int_0^1 (\log x)^3 \cdot \frac{1}{x} dx$

$= \left[\frac{(\log x)^4}{4}\right]_0^1$ $\left(\because \frac{d}{dx} \log x = \frac{1}{x}\right)$

$= \frac{(\log 1)^4}{4} - \frac{(\log 0)^4}{4}$

$= \frac{(0)^4}{4}$ − undefined

This sum is undifined.

Q-11 $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$

$= \int (\tan^{-1} x)^1 \cdot \frac{1}{1+x^2}$ $\left[\int F(x)^n \cdot f'(x) dx = \int \frac{F(x)^{n+1}}{n+1}\right.$

$\left[\frac{(\tan^{-1} x)^2}{2}\right]_0^1$

$= \frac{1}{2} [\tan^{-1} 1)^2 - (\tan^{-1} 0)^2$

$= \frac{1}{2}\left[\left(\frac{\pi}{4}\right)^2 - (0)^2\right]$

$= \frac{1}{2}\left[\left(\frac{\pi}{4}\right)^2\right]$

$= \boxed{\frac{\pi^2}{8}}$ Ans

* Question set for 04 marks

1. $\int \frac{1}{a^2\cos^2 x + b^2\sin^2 x}\,dx$

$= \int \frac{dx}{\cos^2 x\left(a^2 + b^2\frac{\sin^2 x}{\cos^2 x}\right)}$

$\left(\because \frac{1}{\cos\theta} = \sec\theta,\right.$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

$= \int \frac{\sec^2 x \cdot dx}{a^2 + b^2\tan^2 x}$

$= \frac{dy}{b^2\left(\frac{a^2}{b^2} + y^2\right)}$

Put $\tan x = u$

$\sec^2 x = \frac{dy}{dx}$

$\sec^2 x\,dx = dy$

$= \frac{1}{b^2}\int \frac{du}{\left(\frac{a}{b}\right)^2 + y^2}$

$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

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$$= \frac{1}{b^2} \times \frac{1}{a/b} \cdot \tan^{-1} \frac{u}{a/b} + c$$

$$= \frac{1}{b^2} \times \frac{b}{a} \tan^{-1}\left(\frac{bu}{a}\right) + c$$

$$= \boxed{\frac{1}{ab} \tan^{-1}\left(b\frac{\tan x}{a}\right) + c}$$ Ans

Q-3 $\int \frac{\cos 2x}{\cos^2 x \cdot \sin^2 x} dx$

sol^n

$= \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \cdot \sin^2 x} dx$ $\left(\because \cos 2x = \cos^2 x - \sin^2 x \right.$

$= \int \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x} \cdot \sin^2 x} - \frac{\cancel{\sin^2 x}}{\cos^2 x \cdot \cancel{\sin^2 x}} dx$

$= \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} dx$

$= \int \operatorname{cosec}^2 x \, dx - \int \sec^2 x \, dx$

$= \boxed{-\cot x - \tan x + c}$ Ans

Q-4 $\int \frac{1}{x(\log x-1)(\log x-2)}\,dx$

Let $\log x = t$

$\frac{1}{x}\,dx = dt$

$= \int \frac{dt}{(t-1)(t-2)}$

$= \int \frac{dt}{t^2 - 2(t)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 + 2 - \frac{9}{4}}$

$= \int \frac{dt}{\left(t-\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$

$= \boxed{\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c}$

$= \frac{1}{2\left(\frac{1}{2}\right)} \log\left|\frac{t-\frac{3}{2}-\frac{1}{2}}{t-\frac{3}{2}+\frac{1}{2}}\right| + c$

$= \log\left|\frac{t+\left(\frac{-4}{2}\right)}{t+\left(\frac{-2}{2}\right)}\right| + c$

$= \log\left|\frac{t-2}{t-1}\right| + c$

Put the value of t in equation.

$$= \boxed{\log \left| \frac{\log x - 2}{\log x - 1} \right| + C}$$ Ans

Q-5 $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad -(1)$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2 - x)} + \cos(\pi/2 - x)} dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad -(2)$$

Adding (1) and (2)

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Q-6

$$2I = \int_0^{\pi/2} 1 \, dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \pi/2 - 0$$

$$\therefore 2I = \pi/2$$

$$\therefore \boxed{I = \pi/4}$$ Ans

Q-6 $\int_0^{\pi/4} \log(1+\tan x)\,dx$

$$= \int_0^{\pi/4} \log(1+\tan(\pi/4 - x))\,dx$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^{\pi/4} \log\left(1 + \frac{\tan \pi/4 - \tan x}{1+\tan \pi/4 \cdot \tan x}\right)dx$$

$$\left(\because \tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \cdot \tan y}\right)$$

$$= \int_0^{\pi/4} \log\left(1 + \frac{1-\tan x}{1+1\cdot\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log\left(\frac{1+\cancel{\tan x} + 1 - \cancel{\tan x}}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right) dx$$

$$= \int_0^{\pi/4} \log 2 \, dx - \int_0^{\pi/4} \underbrace{(1+\tan x)\, dx}_{I = \text{given}}$$

$$I + I = \cancel{\int_0^{\pi/4}} \log 2 \int_0^{\pi/4} 1 \, dx$$

$$2I = \log 2 \, (x)_0^{\pi/4}$$

$$= \log 2 \left(\frac{\pi}{4} - 0\right)$$

$$= \log 2 \left(\pi/4\right)$$

$$2I = \pi/4 \cdot (\log 2)$$

$$I = \frac{1}{2}\left(\pi/4\right) \log 2$$

$$\boxed{I = \pi/8 \log 2}$$ Ans

श्री स्वामिनारायण डिवाईन मिशन
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Q-7 $\displaystyle\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx$

Sol^n let

$$I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx \quad -(1)$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cot\left(\frac{\pi}{2}-x\right)}}{\sqrt{\tan\left(\frac{\pi}{2}-x\right)}+\sqrt{\cot\left(\frac{\pi}{2}-x\right)}}\,dx$$

$$= \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\cot x}+\sqrt{\tan x}}\,dx \quad -(2)$$

Adding (1) and (2)

$$\therefore 2I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}} + \frac{\sqrt{\tan x}}{\sqrt{\cot x}+\sqrt{\tan x}}\,dx$$

$$\therefore 2I = \int_0^{\pi/2} \frac{\cancel{\sqrt{\cot x}+\sqrt{\tan x}}}{\cancel{\sqrt{\tan x}+\sqrt{\cot x}}}\,dx$$

$$\therefore 2I = \int_0^{\pi/2} 1\,dx$$

$$\therefore 2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0$$

$$2I = \frac{\pi}{2}$$

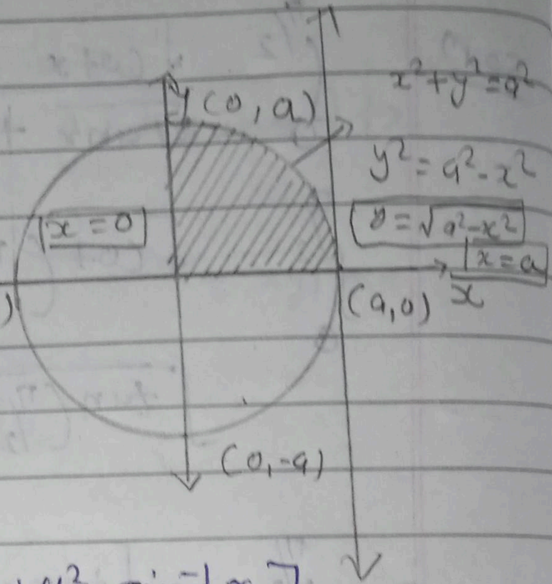
$$\boxed{I = \frac{\pi}{4}}$$ Ans

Meditation is the best ode of worship.

Q-8 Find the area of circle $x^2+y^2=a^2$

Sol^n centre $=(0,0)$

radius $=a$



$$I=\int_a^b y\,dx$$

$$=\int_a^b F(x)\,dx$$

$$=\int_0^a \sqrt{a^2-x^2}\,dx$$

Formula: $\left[\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

$$=\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$=\left[\frac{a}{2}\sqrt{a^2-a^2}+\frac{a^2}{2}\sin^{-1}\frac{a}{a}\right]-\left[\frac{0}{2}\sqrt{a^2-0^2}+\frac{a^2}{2}\sin^{-1}\frac{0}{2}\right]$$

$$=\left[\frac{a}{2}(0)+\frac{a^2}{2}\sin^{-1}(1)\right]-\left[\frac{a^2}{2}(0)\right]$$

$$=\left[0+\frac{a^2}{2}\cdot\frac{\pi}{2}\right]-0$$

$$=\frac{a^2}{2}\cdot\frac{\pi}{2}$$

$$=\frac{a^2\pi}{4}$$

Area $A=4|I|$

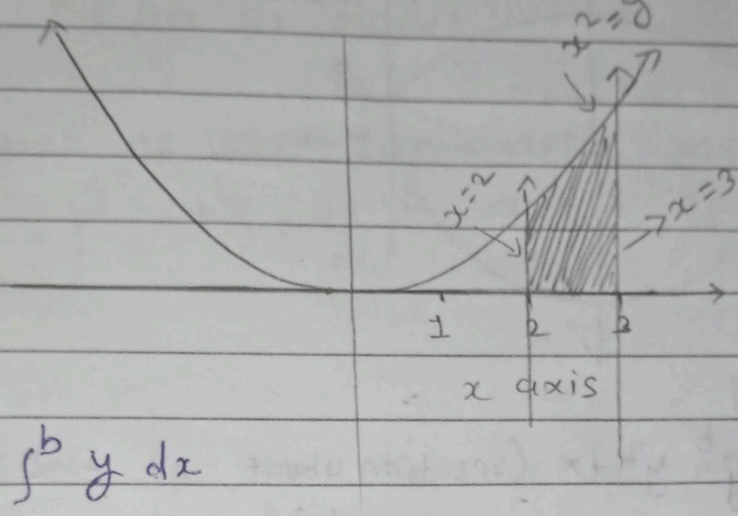
$=\cancel{4}\,\frac{a^2\pi}{\cancel{4}}$

$\boxed{\text{Area}=\pi a^2}$ Ans

Q.9 Find the area enclosed by the parabola $y = x^2$ the x-axis and the line $x = 2$ and $x = 3$.

sol^n $y = x^2$

$x^2 = y$ which is a parabola



$$I = \int_a^b y \, dx$$

$$= \int_a^a F(x) \, dx$$

$$= \int_2^3 x^2 \, dx$$

$$= \left[\frac{x^3}{3}\right]_2^3$$

$$= \frac{1}{3}\left[x^3\right]_2^3$$

$$= \frac{1}{3}\left[(3)^3 - (2)^3\right]$$

$$= \frac{1}{3}\left[27 - 8\right]$$

$$= \frac{1}{3}\left[19\right]$$

$$I = \frac{19}{3}$$

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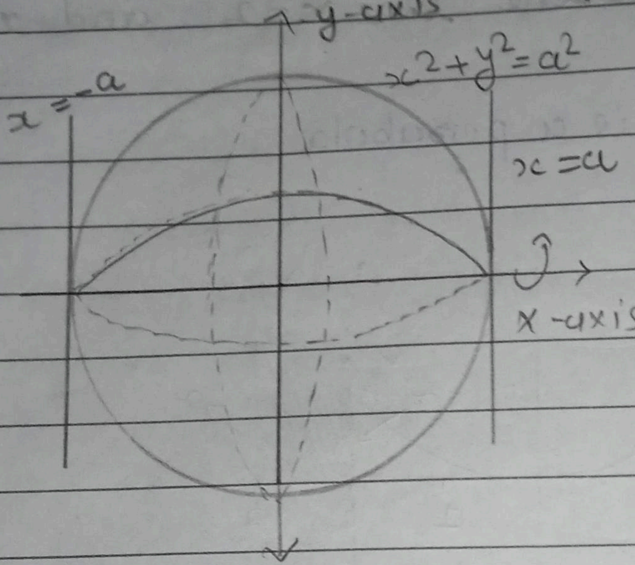
$A = I$

$$\boxed{\text{Area} = \frac{19}{3}}$$ Ans

Q-10 Find the volume of sphere having radius a.

Sol^n



circle $x^2+y^2=a^2$

$y^2=a^2-x^2$

$V=|I|$

$I=\pi\int_a^b y^2\,dx$ (revolution about x-axis)

$=\pi\int_a^b [f(x)]^2\,dx$

$=\pi\int_{-a}^{a}[a^2-x^2]\,dx$

a^2-x^2 is even function

$=2\pi\int_0^a [a^2-x^2]\,dx$

$=2\pi\left[a^2x-\frac{x^3}{3}\right]_0^a$

$=2\pi\left[a^2.a-\frac{a^3}{3}\right)-0$

$=2\pi\left[\frac{3a^3-a^3}{3}\right]$

$=2\pi\left(\frac{2a^3}{3}\right)$

$\boxed{I=\frac{4\pi a^3}{3}}$

volume of sphere=

$V=|I|$

$=\left|\frac{4\pi a^3}{3}\right|$

$=\boxed{\frac{4}{3}\pi a^3}$ Ans