Government Polytechnic, Ahmedabad

Science and Humanities Department

Applied Mathematics

Subject Code: 4320001

Unit-02 Differentiation and Its Applications

[Marks – 16]

Course Outcome (CO b):

- **Demonstrate the ability to solve engineering related problems based on applications of differentiation.**

Important formula of Differentiation

(1) $\frac{d}{dx}k = 0; \quad k \text{ is constant}$

(2) $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

Using above formula

$\rightarrow \frac{d}{dx}\, x = 1$

$\rightarrow \frac{d}{dx}\frac{1}{x} = \frac{d}{dx}x^{-1} = -\frac{1}{x^2}$

$\rightarrow \frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

(3) $\frac{d}{dx}\left(\log_e x\right) = \frac{1}{x}$

(4) $\frac{d}{dx}\left(a^x\right) = a^x \cdot \log_e a$

(5) $\frac{d}{dx}\left(e^x\right) = e^x$

(6) $\frac{d}{dx}\left(\sin x\right) = \cos x$

(7) $\frac{d}{dx}\left(\cos x\right) = -\sin x$

(8) $\frac{d}{dx}\left(\tan x\right) = \sec^2 x$

(9) $\frac{d}{dx}\left(\cot x\right) = -\operatorname{cosec}^2 x$

(10) $\frac{d}{dx}\left(\sec x\right) = \sec x \cdot \tan x$

(11) $\frac{d}{dx}\left(\operatorname{cosec} x\right) = -\operatorname{cosec} x \cdot \cot x$

(12) $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}} \; ; \quad |x| < 1$

(13) $\frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}} \; ; \quad |x| < 1$

(14) $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2} \; ; \quad x \in R$

(15) $\frac{d}{dx}\left(\cot^{-1} x\right) = \frac{-1}{1+x^2} \; ; \quad x \in R$

(16) $\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\sqrt{x^2-1}} \; ; \; |x| > 1$

(17) $\frac{d}{dx}\left(\operatorname{cosec}^{-1} x\right) = \frac{-1}{|x|\sqrt{x^2-1}} \; ; \; |x| > 1$

Working Rules

1. Addition Rule

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

2. Subtraction Rule

$$\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$$

3.

$\rightarrow \frac{d}{dx}(k \cdot u) = k\frac{du}{dx} \quad ; k$ is constant.

4. Multiplication Rule

$\rightarrow \frac{d}{dx}(u \cdot v) = v\frac{du}{dx} + u\frac{dv}{dx}$

$\rightarrow \frac{d}{dx}(u \cdot v \cdot w) = v \cdot w\frac{du}{dx} + u \cdot w\frac{dv}{dx} + u \cdot v\frac{dw}{dx}$

5. Division Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

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Assignment -2

Unit-2 Differentiation and its Applications

* Question Set for 01 Marks:

1. $f(x) = e^{3x}$ then $f'(0) = \underline{3}$

→ $\cancel{\frac{dy}{dx} = e^{3x(1)}}$ $f(x)$ $\frac{dy}{dx} = e^{3x} \cdot 3$

$\cancel{= e^{3}}$ $f'(0) = e^{3(0)} \cdot 3 = e^{0} \cdot 3 = 1 \cdot 3 = 3$

2. $f(x) = 4^{x}$ then $f'(x) = \underline{4^{x} \log_e 4}$

→ $\frac{dy}{dx} = 4^{x}$

$= 4^{x} \log_e 4$

3. $\frac{d}{dx}[\log_e \tan x] = \underline{\sec^2 x}$

→ $\cancel{\frac{d}{dx} = \frac{1}{\tan x} \cdot \sec^2 x}$

$\frac{d}{dx} = \frac{1}{\cancel{e^{\tan x}}} \cdot \cancel{e^{\tan x}} \cdot \sec^2 x$

$= \boxed{\sec^2 x}$

4. $\frac{d}{dx}[\log x^2] = \underline{b)\ 2/x}$

$\frac{dy}{dx} = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$

(5) If $y = x^n$ then $f'(x) = \underline{nx^{n-1}}$

$$\frac{dy}{dx} = x^n \log \frac{d}{dx} x^n$$

$$\boxed{= nx^{n-1}}$$ Ans

(6) If $y = \log(ax+b)$ then $\frac{dy}{dx} = \underline{\frac{a}{ax+b}}$

$$\frac{dy}{dx} = \frac{d}{dx} \log(ax+b)$$

$$= \frac{1}{ax+b} \cdot a(1) + 0$$

$$= \boxed{\frac{a}{ax+b}}$$ Ans

(7) If $\frac{d[\sin^{-1} x + \cos^{-1} x]}{dx} = \underline{0}$

→ (b)

$$\frac{dy}{dx} = \frac{d}{dx} \sin^{-1} x + \frac{d}{dx} \cos^{-1} x$$

$$= \frac{1}{\sqrt{1-x^2}} + \left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = \frac{1-1}{(\sqrt{1-x^2})^2} = \boxed{0}$$

(8) $y = \log(\sin x)$ then $\frac{dy}{dx} =$ $\underline{\cot x}$

→ (b)

$$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x$$

$$= \frac{\cos x}{\sin x}$$

$$= \boxed{\cot x}$$ Ans

(9) If $xy = 1$ then $\frac{dy}{dx} =$ $\underline{-y/x}$

$\therefore xy = 1$

$y = \frac{1}{x}$

taking log of both the sides

$$\log\left(\frac{d}{dx} y\right) = \log\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$\frac{dy}{dx}\frac{1}{y} = \frac{1}{1/x} \cdot \frac{-1}{x^2}$$

$$\frac{1}{y} = \frac{x}{1} \cdot \frac{-1}{x^2}$$

$$\frac{1}{y} = \frac{-1}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{-y}{x}}$$ Ans

10) If $y = \cos x$ then $\frac{d^2y}{dx^2} =$ $\underline{-y}$

$\frac{dy}{dx} = -\sin x$, $\frac{d^2y}{dx^2} = -(\cos x)$

$= \boxed{-y}$ ($\because y = \cos x$ given)

11. If $x = \sin\theta$, $y = \cos\theta$ then $\frac{dy}{dx} = -\tan\theta$

→ (a)

$$\frac{dx}{d\theta} = \cos\theta \quad , \quad \frac{dy}{d\theta} = (-\sin\theta)$$

$$\frac{dy/d\theta}{dx/d\theta} = \frac{dy}{dx} = \frac{-\sin\theta}{\cos\theta} = \boxed{-\tan\theta}$$ Ans

12. $\frac{d\,[x^x]}{dx} = x^x(1+\log x)$

→ (c)

Let $y = x^x$

taking log on both the sides.

$\log y = \log x^x$

$\therefore \frac{1}{y}\frac{dy}{dx} = x \log x$

$\therefore \frac{1}{y}\frac{dy}{dx} = x \frac{d}{dx}\log x + \log x \frac{d}{dx} x$

$= \cancel{x}\frac{1}{\cancel{x}} + \log x \,(1)$

$\therefore \frac{1}{y}\frac{dy}{dx} = 1 + \log x$

$\therefore \frac{dy}{dx} = y(1+\log x)$ $\quad (\because$ given $y = x^x)$

$\therefore \boxed{\frac{dy}{dx} = x^x(1+\log x)}$ Ans

(13) $y = \sin^2 x$ then $\frac{dy}{dx} =$ $\sin 2x$

→ (d)

$y = \sin^2 x$

let $u = \sin x$ $\qquad y = u^2$ $\qquad$ Now

$\frac{dy}{dx} = \cos x$ $\qquad \frac{dy}{du} = 2u$ $\qquad \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 2u \cdot \cos x$

$= 2\sin x \cdot \cos x$

$= \boxed{\sin 2x}$ Ans

* Question set for 03 marks:

1. Using the definition find derivative of $y = \sin x$

Sol:

$f(x) = \sin x$

$f(t) = \sin t$

$$f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x}$$

$$= \lim_{t \to x} \frac{\sin t - \sin x}{t - x}$$

$$= \lim_{t \to x} \frac{2\cos\left(\frac{t+x}{2}\right) \cdot \sin\left(\frac{t-x}{2}\right)}{t - x}$$

$\left[\because S - S = 2CS = 2\cos\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right) \right.$

$$= \lim_{t \to x} \cos\left(\frac{t+x}{2}\right) \cdot \frac{\sin\left(\frac{t-x}{2}\right)}{\frac{t-x}{2}}$$

$\left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \right]$

$$= \lim_{t \to x} \cos\frac{(t+x)}{2} \lim_{t \to x} \frac{\sin\left(\frac{t-x}{2}\right)}{\frac{t-x}{2}}$$

since $t \to x \therefore \frac{t-x}{2} \to 0$

$$= \cos\frac{(x+x)}{2} \cdot 1$$

$$= \cos\frac{\cancel{2}x}{\cancel{2}}$$

$$= \boxed{\cos x}$$ Ans

Q-2 Using the definition find derivative of $y = \sqrt{x}$

here $f(x) = \sqrt{x}$

$f(t) = \sqrt{t}$

$$f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t-x}$$

$$= \lim_{t \to x} \frac{\sqrt{t} - \sqrt{x}}{t-x}$$

$$= \lim_{t \to x} \frac{t^{1/2} - x^{1/2}}{t-x}$$

$$= \cancel{\lim_{t \to x}} \frac{1}{2} x^{\frac{1}{2}-1}$$

$$\left[\because \lim_{x \to a} \frac{x^n - a^n}{x-a} = na^{n-1}\right]$$

$= \frac{1}{2} x^{-1/2}$

$= \boxed{\frac{1}{2\sqrt{x}}}$ Ans

Q(3) $y = \frac{a+b\sin x}{b+a\sin x}$ find $\frac{dy}{dx}$

Differentiating with respect to x

$\frac{dy}{dx} = \frac{b+a\sin x \frac{d}{dx}(a+b\sin x) - (a+b\sin x)\frac{d}{dx}(b+a\sin x)}{(b+a\sin x)^2}$ (∵ using division rule)

$= \frac{b+a\sin x \cdot (0+b\cos x) - (a+b\sin x)(0+a\cos x)}{(b+a\sin x)^2}$

(∵ using Addition rule)

$= \frac{b+a\sin x\,(b\cos x) - (a+b\sin x)(a\cos x)}{(b+a\sin x)^2}$

$= \frac{b\cos x\,(b+a\sin x) - a\cos x\,(a+b\sin x)}{(b+a\sin x)^2}$

$= \frac{\cos x\,(b^2 + ab\cancel{\sin x} - a^2 - ab\cancel{\sin x})}{(b+a\sin x)^2}$

$$\frac{dy}{dx} = \frac{\cos x(b^2 - a^2)}{(b + a\sin x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{(b^2 - a^2)\cos x}{(a\sin x + b)^2}}$$ Ans

Q-4 $x + y = \sin(xy)$ find $\frac{dy}{dx}$

Sol^n Differentiating with respect to x

$$\frac{dy}{dx} = \frac{d}{dx}x + \frac{dy}{dx} = \cos xy\left(x\frac{d}{dx}y + y\frac{d}{dx}x\right)$$

(using multiplication rule)

$$1 + \frac{dy}{dx} = \cos xy\left(x\frac{dy}{dx} + y(1)\right)$$

$$1 + \frac{dy}{dx} = x\cos xy\frac{dy}{dx} + y\cos xy$$

$$\frac{dy}{dx} - x\cos xy\frac{dy}{dx} = y\cos xy - 1$$

$$\therefore \frac{dy}{dx}(1 - x\cos xy) = y\cos xy - 1$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{y\cos xy - 1}{1 - x\cos xy}}$$ Ans

Q-5 Find $\frac{dy}{dx}$; $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$

$\frac{dx}{d\theta} = a(q + \cos\theta)$, $\frac{dy}{d\theta} = a(0 - (-\sin\theta))$

$\frac{dy}{d\theta} = a(\sin\theta)$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a(1+\cos\theta)}$$

$$= \frac{2\sin\theta/2 \cdot \cos\theta/2}{2\cos^2\theta/2}$$

$$\left[\because \sin\theta = 2\sin\theta/2 \cdot \cos\theta/2, \quad 1 + \cos\theta = 2\cos^2\theta/2 \right)$$

$$= \frac{\cancel{2}\sin\theta/2 \cdot \cancel{\cos\theta/2}}{\cancel{2} \cdot \cos\theta/2 \cdot \cancel{\cos\theta/2}}$$

$$= \frac{\sin\theta/2}{\cos\theta/2}$$

$$\frac{dy}{dx} = \boxed{\tan\theta/2}$$ **Ans**

Q-6 $y = (\sin x)^x$, Find $\frac{dy}{dx}$

taking log on both the sides

$\log y = \log (\sin x)^x$

$\therefore \log y = x \log(\sin x)$

Differentiating with respect to x

$\therefore \frac{1}{y}\frac{dy}{dx} = x \frac{d}{dx} \log(\sin x) + \log(\sin x) \frac{d}{dx} x$

$\therefore \frac{1}{y}\frac{dy}{dx} = x \frac{1}{\sin x} \cdot \cos x + \log(\sin x)(1)$

$\therefore \frac{1}{y}\frac{dy}{dx} = x \frac{\cos x}{\sin x} + \log(\sin x)$

$\therefore \frac{1}{y}\frac{dy}{dx} = x \cot x + \log(\sin x)$

$\therefore \frac{dy}{dx} = y[x \cot x + \log(\sin x)]$

$\therefore \boxed{\frac{dy}{dx} = (\sin x)^x [x \cot x + \log(\sin x)]}$ Ans.

Q-7. Differentiate $\sin^{-1}x$ with respect to $\cos^{-1}x$

Let $y_1 = \sin^{-1}x$ $y_2 = \cos^{-1}x$

$\frac{dy_1}{dx} = \frac{d}{dx} \sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

$$\frac{dy_1}{dx} = \frac{d}{dx}\cos^{-1}x$$

$$= \frac{-1}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy_1}{dy_2} = \frac{\frac{1}{\cancel{\sqrt{1-x^2}}}}{\frac{-1}{\cancel{\sqrt{1-x^2}}}}$$

$$= \frac{1}{-1}$$

$$\cancel{dt} = \boxed{-1} \text{ Ans}$$

Q-8 $y = ae^{kx} + be^{-kx}$ prove that $\frac{d^2y}{dx^2} = k^2y$ where a, b, k are constants.

Sol^n $\therefore$ Differentating with respect to x

$$\therefore \frac{dy}{dx} = ae^{kx}\cdot k(1) + be^{-kx}\cdot(-k)(1)$$

$$= ae^{kx}\cdot k + be^{-kx}\cdot(-k)$$

$$= kae^{kx} - kbe^{-kx}$$

$$= k(ae^{kx} - be^{-kx})$$

$$\frac{d^2y}{dx^2} = k(a\cdot e^{kx}\cdot k - be^{-kx}\cdot(-k))$$

$$= k^2(ae^{kx} + be^{-kx})$$

Prove $\boxed{\frac{d^2y}{dx^2} = k^2y}$ Ans

Q-9 $y = a\cos pt + b\sin pt$ prove that $\frac{d^2x}{dy^2} + p^2y = 0$

sol^n Let $y = u + v$

$\frac{du}{dt} = \frac{d}{dx}(a\cos pt)$

$= a(-\sin pt) \cdot p(1)$

$= -\sin pt \cdot p$

$= -p\sin pt$

$\frac{dv}{dt} = \frac{d}{dx} b\sin pt$

$= b\cos pt \cdot p(1)$

$= b\cos pt \cdot p$

$= pb\cos pt$

$\frac{dy}{dx} = \cancel{u+} - p\sin pt + pb\cos pt$

$= p(-\sin pt + b\cos pt)$

$\boxed{\frac{dy}{dx} = p(-\sin pt + b\cos pt)}$ — L.H.S

$\frac{d^2y}{dx^2} = p(-a\cos pt \cdot p + b(-\sin pt) \cdot p)$

$= p(-pa\cos pt - pb\sin pt)$

$= p^2(-a\cos pt - b\sin pt)$

$= -p^2(a\cos pt + b\sin pt)$

$= -p^2 y$ — R.H.S

given equation

$\therefore \frac{d^2y}{dx^2} + p^2y = 0$

$\therefore \frac{d^2y}{dx^2} = -p^2y$

Prove

$\boxed{L.H.S = R.H.S}$

Q-1° If $s = 2t^3 - 3t^2 - 12t + 5$ then find v and a at $t = 2$ sec

Sol^n → velocity $(v) = \frac{ds}{dt} = 2(3t^2) - 3(2t) - 12(1) + 0$

$= 6t^2 - 6t - 12$ — ①

velocity at any time t

→ $a = \frac{dv}{dt} = 6(2t) - 6(1) - 0$

$= 12t - 6$ — ②

a at any time t

$v = \left[\frac{ds}{dt}\right]_{t=2sec}$

$= 6(2)^2 - 6(2) - 12$

$= 6(4) - 12 - 12$

$= 24 - 24$

= ~~0 distance~~ /

$= \boxed{0 \text{ unit/sec}}$ Ans

$a = \left[\frac{dv}{dt}\right]_{t=2sec}$ $= 12(2) - 6$

$= 24 - 6$

$= \boxed{18 \text{ unit/sec}^2}$ Ans

* Question set for 04 marks.

1. If $y = \log(x + \sqrt{1+x^2})$ then prove that

$$(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$$

sol^n Differentiating with respect to x.

here $y = \log(x + \sqrt{1+x^2})$

$$\therefore \frac{dy}{dx} = \frac{1}{x+\sqrt{1+x^2}} \cdot \left(1 + \frac{1}{2\sqrt{1+x^2}} \cdot 2x\right)$$

$$= \frac{1}{x+\sqrt{1+x^2}} \cdot \left(1 + \frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}}\right)$$

$$= \frac{1}{\cancel{x+\sqrt{1+x^2}}} \cdot \frac{\cancel{\sqrt{1+x^2}+x}}{\sqrt{1+x^2}}$$

$$= \frac{1}{\sqrt{1+x^2}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$$

$$\sqrt{1+x^2}\,\frac{dy}{dx} = 1$$

$$\therefore \left(\sqrt{1+x^2}\right)^2 \cdot \left(\frac{dy}{dx}\right)^2 = (1)^2$$

$$1+x^2\left(\frac{dy}{dx}\right)=1$$

$$\therefore 1+x^2\cdot 2\left(\frac{dy}{dx}\right)\cdot\frac{d^2y}{dx^2}+2x\left(\frac{dy}{dx}\right)^2=0$$

Prove $\therefore (1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$

(2) If $x=\sin t$, $y=\sin pt$ then prove that

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$$

$\frac{dy}{dt}=\frac{d}{dx}\sin pt$ $\qquad$ $\frac{dx}{dt}=\frac{d}{dx}\sin t$

$=\cos pt\cdot p(1)$ $\qquad$ $=\cos t$

$= p\cos pt$

$$\frac{\frac{dy}{dt}}{dx/dt}=\frac{dy}{dx}=\frac{\cancel{\cos t}}{p\cancel{\cos pt}}\quad\frac{p\cos pt}{\cos t}$$

$$\left(\frac{dy}{dx}\right)^2=\frac{p^2\cos^2 pt}{\cos^2 t}$$

$$=\frac{p^2(1-\sin^2 pt)}{1-\sin^2 t}$$

$$=\frac{p^2(1-y^2)}{1-x^2}$$ $\quad(\because \sin pt=y,\ \sin t=x)$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = p^2(1-y^2)$$

$$\therefore (1-x^2)\, 2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + (-2x)\left(\frac{dy}{dx}\right)^2 =$$

$$p^2\left(-2y\frac{dy}{dx}\right)$$

$$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -p^2y$$

(Dividing by $2\frac{dy}{dx}$)

prove

$$\therefore \boxed{(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0}$$

(3) If $S = t^3 - 6t^2 + 9t + 4$ then

i) find s and a at v=0

ii) find s and v at a=0

sol^n → i) velocity = $\frac{ds}{dt}$

$$0 = 3t^2 - 6(2t) + 9(1) + 0$$

$$0 = 3t^2 - 12t + 9$$

$$0 = 3(t^2 - 4t + 3)$$

$$t^2 - 4t + 3 = 0/3$$

$$\therefore t^2 - 4t + 3 = 0$$

$$\therefore (t-3)(t-1) = 0$$

$$\therefore t = 3 \quad or \quad t = 1$$

Accelaration = $\left[\frac{dv}{dt}\right]_{t=3}$ $= 3(2t) - 12(1) + 0$

$= 6t - 12$

$= 6(3) - 12$

$= 18 - 12$

$= \boxed{6 \text{ distunce/time}^2}$ Ans

Acceletation = $\left[\frac{dv}{dt}\right]_{t=1}$

$= 6t - 12$

$= 6(1) - 12$

$= 6 - 12$

$= \boxed{-6 \text{ distunce/time}^2}$ Ans

$S_{t=3} = (3)^3 - 6(3)^2 + 9(3) + 4$

$= 27 - 6(9) + 27 + 4$

$= \cancel{27} - 54 + \cancel{27} + 4 = \boxed{4 \text{ unit}}$

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$$S_{t=1} = (1)^3 - 6(1)^2 + 9(1) + 4$$
$$= 1 - 6 + 9 + 4$$
$$= 14 - 6$$
$$= \boxed{8 \text{ unit}}$$

→ ii) Accelaration $= 6t - 12 = 0$

$$\therefore 6(t-2) = 0$$
$$\therefore t = 2$$
$$\therefore \boxed{t = 2}$$

$$v = \left[\frac{ds}{dt}\right]_{t=2} = 3(2)^2 - 12(2) + 9$$
$$= 3(4) - 24 + 9$$
$$= 12 - 24 + 9$$
$$= 21 - 24$$
$$= \boxed{-3 \text{ distance/ time}}$$ Ans

$$S_{t=2} = (2)^3 - 6(2)^2 + 9(2) + 4$$
$$= 8 - 6(4) + 18 + 4$$
$$= 8 - 24 + 18 + 4$$
$$= 30 - 24$$
$$= \boxed{6 \text{ unit}}$$ Ans

(4) The equation of motion of particle $s = t^3 - 5t^2 + 3t + 7$ when the particle come to rest ? Find its acceleration at that time.

Sol^n

here $s = t^3 - 5t^2 + 3t + 7$

velocity $v = \frac{ds}{dt} = 3t^2 - 5(2t) + 3(1) + 0$
$= 3t^2 - 10t + 3$

Now, $v = 0$ when the particle comes to rest

$\therefore \quad 3t^2 - 10t + 3 = 0$

$\therefore \quad 3t^2 - 9t - 1t + 3 = 0$

$\therefore \quad 3t(t-3) - 1(t-3) = 0$

$\therefore \quad (3t-1)(t-3) = 0$

$\therefore \quad \boxed{t = 3}$ or $\boxed{t = \frac{1}{3}}$

Acceleration $(a) = \frac{dv}{dt} = 3(2t) - 10(1) + 0$
$= 6t - 10$

$a = \left[\frac{dv}{dt}\right]_{t=3} = 6(3) - 10$
$= 18 - 10$
$= \boxed{8 \text{ distance / time}^2}$ Ans

$a = \left[\frac{dv}{dt}\right]_{t=1/3} = \cancel{6}2(\frac{1}{\cancel{3}}) - 10$

$= 2 - 10$

$= \boxed{-8 \text{ distance / time}^2}$ Ans

(5) Find the maxima and minima at the function.

a) $f(x) = 2x^3 - 15x^2 + 36x + 10$

Soln $y = f(x) = 2x^3 - 15x^2 + 36x + 10$

$\frac{dy}{dx} = 2(3x^2) - 15(2x) + 36(1) + 0$

$= 6x^2 - 30x + 36$

Now $\frac{dy}{dx} = 0$

$6x^2 - 30x + 36 = 0$

$\therefore 6(x^2 - 5x + 6) = 0$

$\therefore x^2 - 5x + 6 = 0/6$

$\therefore x^2 - 5x + 6 = 0$

$\therefore (x-3)(x-2) = 0$

$\therefore \boxed{x = 3}$ or $\boxed{x = 2}$

Now $\frac{d^2y}{dx^2} = 6(2x) - 30(1) + 0$

$= 12x - 30$

For $x = 3$, $\left[\frac{d^2y}{dx^2}\right]_{x=3} = 12(3) - 30$

$= 36 - 30$

$= 6 > 0$

$\boxed{x = 3 \text{ is minima}}$

minimum value $= 2(3)^3 - 15(3)^2 + 36(3) + 10$

$= 2(27) - 15(9) + 108 + 10$

$= 54 - 135 + 108 + 10$

$= 172 - 135$

$\boxed{\text{minimum value} = 37}$

For $x = 2$

$$\left[\frac{d^2y}{dx^2}\right]_{x=2} = 12(2) - 30$$
$$= 24 - 30$$
$$= -6 < 0$$

$\boxed{x = 2 \text{ is maxima}}$

maximum value $= 2(2)^3 - 15(2)^2 + 36(2) + 10$
$= 2(8) - 15(4) + 72 + 10$
$= 16 - 60 + 72 + 10$
$= 98 - 60$

$\boxed{\text{maximum value} = 38}$

(b) $f(x) = x^3 - 4x^2 + 5x + 7$

Soln $y = f(x) = x^3 - 4x^2 + 5x + 7$

$$\frac{dy}{dx} = 3x^2 - 4(2x) + 5(1) + 0$$
$$= 3x^2 - 8x + 5 + 0$$
$$= 3x^2 - 8x + 5$$

Now

$$\frac{dy}{dx} = 0$$

$\therefore 3x^2 - 8x + 5 = 0$

$\therefore (3x - 5)(x - 1) = 0$

$\therefore x = 1$ or $x = 5/3$

Now $\frac{d^2y}{dx^2} = 3(2x) - 8(1) + 0$
$= 6x - 8$

For

$$\left[\frac{d^2y}{dx^2}\right]_{x=1} = 6(1) - 8$$
$$= 6 - 8 = -2 < 0$$

→ $x = 1$ is maxima

maximum value $= (1)^3 - 4(1)^2 + 5(1) + 7$

$= 1 - 4 + 5 + 7$

$= 13 - 4$

$= 9$

→ for $\left[\frac{d^2y}{dx^2}\right]_{x=5/3} = 6\left(\frac{5}{3}\right) - 8$

$= 10 - 8$

$= 2 > 0$

$$\boxed{x = \frac{5}{3} \text{ is minima}}$$

minimum value $= \left(\frac{5}{3}\right)^3 - 4\left(\frac{5}{3}\right)^2 + 5\left(\frac{5}{3}\right) + 7$

$= \frac{125}{27} - 4\left(\frac{25}{9}\right) + \frac{25}{3} + 7$

$= \frac{125}{27} - \frac{100}{9} + \frac{25}{3} + \frac{7}{1}$

$= \frac{125 - 300 + 225 + 189}{27}$

$$\boxed{\text{minimum value} = \frac{239}{27}}$$

(c)

$f(x) = x^3 - 3x + 11$

$y = f(x) = x^3 - 3x + 11$

$\frac{dy}{dx} = 3x^2 - 3(1) + 0$

$= 3x^2 - 3$

Now $\frac{dy}{dx} = 0$

$3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$x^2 - 1 = 0$

$(x + 1)(x - 1) = 0$

$x = -1, x = 1$

Now

$\frac{d^2y}{dx^2} = 3(2x) - 0$

$= 6x$

For $\left[\frac{d^2y}{dx^2}\right]_{x=-1} = 6(-1)$

$= -6 < 0$

$\boxed{x = -1 \text{ is maxima}}$

maximum value $= (-1)^3 - 3(-1) + 11$

$= -1 + 3 + 11$

$= -1 + 14$

$= \boxed{13}$

For $\left[\frac{d^2y}{dx^2}\right]_{x=1} = 6(1)$

$= 6 > 0$

$\boxed{x = 1 \text{ is minima}}$

minimum value $= (1)^3 - 3(1) + 11$

$= 1 - 3 + 11$

$= 12 - 3$

$= \boxed{9}$

(d) $f(x) = x + \frac{1}{x}$

Sol^n:- $F(x) = x + \frac{1}{x}$

$\frac{dy}{dx} = 1 + \left(\frac{-1}{x^2}\right)$

$= 1 - \frac{1}{x^2}$

$= \frac{x^2 - 1}{x^2}$

$\frac{d^2y}{dx^2} = 0 - \left(\frac{-1}{(x^2)^2} \cdot 2x\right)$

$= 0 + \frac{1}{x^4} \cdot 2x$

$\frac{d^2y}{dx^2} = \frac{2}{x^3}$

$\frac{dy}{dx} = 0$

$\therefore \frac{x^2 - 1}{x^2} = 0$

$\therefore x^2 - 1 = 0$

$\therefore \boxed{x = \pm 1}$

Now $\left[\frac{d^2y}{dx^2}\right]_{x=-1} = \frac{2}{(-1)^3}$

$= \frac{2}{-1}$

$= -2 < 0$

$\boxed{x = -1 \text{ is maxima}}$

maximum value =

$-1 + \frac{1}{-1}$

$-1 + (-1)$

$-1 - 1$

$= \boxed{-2}$

Now $\left[\frac{d^2y}{dx^2}\right]_{x=1} = \frac{2}{1^3}$

$= 2 > 0$

$\boxed{x = 1 \text{ is minima}}$

minimum value

$1 + \frac{1}{1}$

$= \boxed{2 \text{ is maximum value}}$

(c) $f(x) = \sin x + \cos x$

sol^n $f(x) = \sin x + \cos x$

$\frac{dy}{dx} = \cos x - \sin x$

$\frac{d^2y}{dx^2} = -\sin x - \cos x$

$= -(\sin x + \cos x)$

$\frac{dy}{dx} = 0$

$\cos x - \sin x = 0$

$\therefore \cos x = \sin x$

$\therefore x = \pi/4, \; x = 5\pi/4$

for $x = \pi/4$

$\left[\frac{d^2y}{dx^2}\right]_{x=\pi/4} = -(\sin \pi/4 + \cos \pi/4)$

$= -\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$

$= -\frac{2}{\sqrt{2}}$

$= -\sqrt{2} < 0$

$\boxed{x = \pi/4 \text{ maxima.}}$

maximum value $= \sin \pi/4 + \cos \pi/4$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \boxed{\sqrt{2}}$

for $x = 5\pi/4$

$\left[\frac{d^2y}{dx^2}\right]_{x=5\pi/4} = -(\sin 5\pi/4 + \cos 5\pi/4)$

$= -\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$

$= +\sqrt{2} > 0$

$\boxed{x = 5\pi/4 \text{ is minima}}$

minimum value $= \sin\left(\frac{5\pi}{4}\right) + \cos 5\pi/4$

$= \frac{-1}{\sqrt{2}} + \left(-\frac{1}{\sqrt{2}}\right)$

$= -\frac{2}{\sqrt{2}}$

$= \boxed{-\sqrt{2}}$ minimum value

(6) $f(x)$ = If $\sin y = x \sin(a+y)$ then prove that $\sin a \frac{dy}{dx} = \sin^2(a+y)$

$\sin y = x\sin(a+y)$

$$\boxed{x = \frac{\sin y}{\sin(a+y)}}$$

$\therefore y = f(x)$

$\frac{dy}{dx} = f'(x)$

$\frac{dx}{dy}$, $x = f(y)$

$\frac{dx}{dy} = f'(y)$

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y\cos(a+y)}{\sin^2(a+y)}$$

(using division rule)

$$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)}$$

$(\because \sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B)$

here, $\sin(A-B) = \sin(a+y-y)$

$$\frac{dx}{dy} = \frac{\sin a}{\sin^2(a+y)}$$

$$\boxed{\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}}$$ prove

Dt: / / Pg.no.:

(7) If $y = a(1-\cos\theta)$, $x = a(\theta+\sin\theta)$ then prove that $\frac{d^2y}{d\theta^2} = \frac{1}{4a}\sec^4\theta/2$

Solⁿ

$y = a(1-\cos\theta)$

$\frac{dy}{d\theta} = a(0-(-\sin\theta))$

$= a\sin\theta$

$x = a(\theta+\sin\theta)$

$\frac{dx}{d\theta} = a(1+\cos\theta)$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$= \frac{\cancel{a}\sin\theta}{\cancel{a}(1+\cos\theta)}$$

$$= \frac{\sin\theta}{1+\cos\theta}$$

$\sin2\theta = 2\sin\theta\cdot\cos\theta$

$\sin\theta = 2\sin\theta/2\cdot\cos\theta/2$

(A)

$\cos2\theta = 2\cos^2\theta - 1$

$1+\cos2\theta = 2\cos^2\theta$

$\cos\theta = 2\cos^2\theta/2 - 1$

$\cos\theta + 1 = 2\cos^2\theta/2$

(B)

$$= \frac{\cancel{2}\sin\theta/2\cdot\cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$$

$$= \frac{\sin\theta/2}{\cos\theta/2}$$

$$\frac{dy}{dx} = \boxed{\tan\theta/2}$$

$$\frac{d^2y}{dx^2} = \sec^2\theta/_2 \cdot \frac{1}{2} \cdot \frac{d\theta}{dx}$$

$$= \frac{1}{2}\sec^2\theta/_2 \cdot \frac{1}{a(1+\cos\theta)}$$

$$= \frac{1}{2a}\sec^2\theta/_2 \cdot \frac{1}{2\cos^2\theta/_2}$$

$$= \frac{1}{4a}\sec^2\theta/_2 \cdot \sec^2\theta/_2$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{1}{4a}\sec^4\theta/_2}$$ Ans